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Finding the Optimal Placement of Evacuation Centers by Antibase Set of Intuitionistic Fuzzy Graph

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***Abstract*: The problem of choosing places for evacuation centers is considered in this paper. We consider the case when the territory model is represented by an intuitionistic fuzzy graph. To solve this problem, the concept of a minimal antibase of such graph is introduced, and on its basis, the concept of an antibase set as an invariant of this graph is introduced too. A method and algorithm for calculating the minimal antibases are proposed and justified. The problem of finding all minimal antibases of the graph allows us to solve the task of determining the antibase set. The paper considers a numerical example of finding the antibase set of an intuitionistic fuzzy graph. The task of choosing the places of evacuation centers in an optimal way depends on their number. The calculation of the minimal antibase set allows us to directly solve this problem.**

Keywords: Evacuation, Evacuation Centers, Intuitionistic Fuzzy Graph, Intuitionistic Fuzzy Path, Minimal Intuitionistic Antibase Vertex Subset, Antibase Set.

I. Introduction

The evacuation of the population is one of the effective ways to protect the population, material and cultural values from the dangers arising from natural and man-made emergencies. The essence of evacuation is the organized movement of the population and material and cultural values to safe areas. In many situations, this method is the only acceptable method of protection, for example, in the event of catastrophic flooding, long-term radioactive contamination of the area with densities higher than permissible, etc.

Significant volumes, the complexity of organizing and conducting evacuation measures impose increased requirements on the optimal placement of evacuation centers, their timely and high-quality preparation for the evacuation of the population. We are talking about places of collection, reception, intermediate evacuation centers and optimization of evacuation routes.

A decision maker (DM), with a comprehensive assessment of the circumstances, deals with many factors and uncertainties: the level of danger, the behavior of the population, the location of evacuation centers, transport.

To facilitate the management of evacuation operations, evacuation plans should be developed in advance during the preparation phase:

- for stakeholders involved in crisis management;
- for events in various evacuation scenarios; for escape routes, shelters and behavior of people;
- to manage shelters and resource providers;
- to return after evacuation.

The work [1] presents various methods for support effective evacuation planning. However, both a general evacuation planning model and a general set of specific parameters that should be included in the plan as initial data are missing here. The work [2] considers various stages in the planning of flood evacuation. But there is no approach to assessing information about the current situation to justify the need for evacuation. The evacuation studies carried out in [3] identified the following tasks for the development of an evacuation plan at the preparation stage: determining the predicted parameters and disaster scenarios, characterizing the vulnerability, determining actions and data such as the capacity of the transport network, the number of evacuees, strategies and evacuation scenarios, their optimization, selection of an evacuation plan and its application in real time. The work [4] presents a program for modeling floods, traffic flows during evacuation, as well as optimization of possible strategies. According to their purpose, evacuation modeling tools can be divided into two types: models of specific disasters [5, 6] and models that provide evacuation [1, 7-9].

Existing evacuation traffic models can be classified as:

- flow models [10];
- agent-based models, in which individual vehicles are considered as agents with autonomous behavior interacting with other vehicles [11];
- scenario-based simulation models to identify evacuation bottlenecks [12].

The paper [13] presents a review of the literature on the methods of mathematical modeling of evacuation traffic. Time models taking into account critical paths are presented in [14, 15].

The models listed above are commonly used as support tools for evacuation planning using geographic information systems (GIS). The use of GIS for evacuation planning makes it possible to display the results of evacuations on maps, which makes it easier for decision makers to understand. The integration of GIS technologies, simulation models and 3D visualization for traffic impact analysis appears to open up new perspectives for evacuation planning and decision making.

The decision to initiate a mass evacuation plan based on a crisis assessment becomes a challenge for decision makers. Several issues related to this problem are considered in the literature: criteria for making decisions about evacuation, the decision-making process taking into account uncertain factors, as well as decision-making modeling [16, 17].

The main factors considered by existing methods and models for making decisions about evacuation are:
- hazard forecast;
- danger alert;
- assessment of the consequences of the disaster;
- evacuation time;
- evacuation costs;
- factors of uncertainty.

Accounting for forecast uncertainty is a complex part of the decision making. Several studies have been conducted to quantify the uncertainty of possible developments and to help decision makers determine what to plan for. Some studies emphasize the importance of interpreting uncertainty in predicting the level of danger and evacuation [18, 19]. These studies illustrate how different levels of forecast uncertainty affect the optimal evacuation decision over time.

The assessment of natural hazards is always subject to uncertainty due to the lack of accurate knowledge, the complexity of physical processes, and their natural variability. Therefore, many studies emphasize the importance of interpreting uncertainty in predicting the local level of danger and in the perspective of evacuation.

Subjective uncertainty factors are not widely represented in the literature. They are difficult to model, so studies are required to consider subjective uncertainty in the process of planning to evacuate. At present, the need to model support for making decisions about evacuation is becoming increasingly important. Such tasks are difficult to formalize, characterized by incompleteness and fuzziness of the initial information, fuzziness of the goals set [20, 21].

Several methods have been proposed to solve them under fuzzy conditions, and fuzzy set theory has been extended by developing new types of fuzzy sets, such as non-stationary fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, oscillating fuzzy sets. Fuzzy logic, dealing with subjective uncertainty, in many cases turns out to be more effective than using only deterministic, probabilistic, or heuristic approaches. In addition, the application of the theory of fuzzy sets and fuzzy logic to solving the problems of evacuation, choosing the location of evacuation centers allows you to include in the decision-making model data that are not always quantifiable, as well as incomplete, inaccessible information, and partially ignored facts. Therefore, fuzzy logic methods are particularly suitable for making evacuation decisions when there is little data, knowledge of cause-and-effect relationships is inaccurate, and observations and criteria can be expressed in linguistic qualitative terms.

This paper considers one of the tasks that arises when supporting decision-making during evacuation, namely, the choice of locations for evacuation centers on the plan of a certain territory. At the same time, the territory model is represented by an intuitionistic fuzzy graph. In the graph under consideration, the vertices determine the locations of people and the possible locations of evacuation centers, and the intuitionistic degree assigned to the edges determines the degree of safety of movement along this edge. Concepts of the minimal antibase and the antibase set of intuitionistic fuzzy graph are introduced here. It is shown that the choice of the best placement of evacuation centers is equivalent to finding an intuitionistic fuzzy set of antibases for a given graph.

II. Preliminaries

The concept of a fuzzy set as a method of representing uncertainty was proposed and discussed in [22]. In the articles [23, 24], the fuzzy set was generalized as the concept of an intuitionistic fuzzy set. In the latter, the degree of non-membership was added to the concept of the membership function of the fuzzy set.

The original definition of a fuzzy graph [25] was based on the concept of a fuzzy relationship between vertices [26]. The concept of complementing a fuzzy graph and some operations on fuzzy graphs were considered in [27, 28]. The concepts of an intuitionistic fuzzy relation and an intuitionistic fuzzy graph were considered in the papers [29, 30]. The concepts of a dominating set, and a base set as invariant of intuitionistic fuzzy graph were introduced in the papers [31 - 35].

The intuitionistic fuzzy set on the set X is the set of triples $\tilde{A}=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\}$ [23]. Here $\mu_A(x)\in[0,1]$ is the membership function of x in $\tilde{A}$, and $\nu_A(x)\in[0, 1]$ is the non-membership function x in $\tilde{A}$. Moreover, for any $x\in X$ the values $\mu_A(x)$, and $\nu_A(x)$ must satisfy the condition:

$$\mu_A(x) + \nu_A(x) \le 1.$$

The intuitionistic fuzzy relation $\tilde{R}=(\mu_R(x,y),\nu_R(x,y))$ on the set $X\times Y$ is the set $\tilde{R}=\{\langle(x,y), \mu_R(x,y), \nu_R(x,y)\rangle \mid (x,y)\in X\times Y\}$, where μ_R: $X\times Y\to[0,1]$ and ν_R: $X\times Y\to[0,1]$. In this case, the following condition is fulfilled:

$$(\forall x,y\in X)[\mu_R(x,y) + \nu_R(x,y) \le 1].$$

Let $p=(\mu(p),\nu(p))$ and $q=(\mu(q),\nu(q))$ be intuitionistic fuzzy variables, where $\mu(p)+\nu(p)\le 1$ и $\mu(q)+\nu(q)\le 1$. Then the operations “&” and “∨” are defined as [24]:

$$p\&q = (\min(\mu(p), \mu(q)), \max(\nu(p), \nu(q))), \tag{1}$$

$$p \vee q = (\max(\mu(p), \mu(q)), \min(\nu(p), \nu(q))). \quad (2)$$

We will consider $p \leq q$ if $\mu(p) \leq \mu(q)$ and $\nu(p) \geq \nu(q)$. Otherwise, we will assume that p and q are incommensurable intuitionistic fuzzy variables.

An intuitionistic fuzzy graph [29,30] is a pair $\tilde{G} = (\tilde{A}, \tilde{U})$, where $\tilde{A} = \langle V, \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy set on the vertex set V, $\tilde{U} = \langle V \times V, \mu_U, \nu_U \rangle$ is an intuitionistic fuzzy set of edges, and the following inequalities hold:

$$\mu_U(xy) \leq \min(\mu_A(x), \mu_A(y)); \quad (3)$$

$$\nu_U(xy) \leq \max(\nu_A(x), \nu_A(y)); \quad (4)$$

$$(\forall x, y \in V)[0 \leq \mu_U(xy) + \nu_U(xy) \leq 1]. \quad (5)$$

III. Antibase Set

Let $\tilde{G} = (\tilde{A}, \tilde{U})$ be an intuitionistic fuzzy graph. Let $p(x,y) = (\mu(x,y), \nu(x,y))$ be an intuitionistic fuzzy variable that determines the degree of adjacency and degree of non-adjacency of vertex y from vertex x.

An intuitionistic fuzzy path $\tilde{L}(x_i, x_j)$ [36, 37] from a vertex x_i to a vertex x_j of a graph $\tilde{G} = (\tilde{A}, \tilde{U})$ is a directed sequence of vertices and edges in which the end vertex of any edge (except for x_j), is the starting vertex of the next arc.

The strength of the path $s(\tilde{L}(x_i, x_j))$ is determined by the smallest value of the degrees of vertices and edges included in this path [38]. Taking into account expressions (3) and (4), the strength $s(\tilde{L}(x_i, x_j))$ of the path $\tilde{L}(x_i, x_j)$ is determined only by the values of its edges:

$$s(\tilde{L}(x_i, x_j)) = \underset{(x_\alpha, x_\beta) \in \tilde{L}(x_i, x_j)}{\&} p(x_\alpha, x_\beta).$$

Here the operation & is defined according to expression (1).

Since the strength of the path depends on the intuitionistic degrees of the edges and does not depend on the degrees of the vertices, we will further consider intuitionistic fuzzy graphs with crisp vertices: $\tilde{G} = (V, \tilde{U})$.

The vertex x_j is *reachable* from the vertex x_i if there exists an intuitionistic fuzzy path $\tilde{L}(x_i, x_j)$ with degree $s\left(\tilde{L}(x_i, x_j)\right)$ different from (0,1). Each vertex x_i is considered to be reachable from itself with degree $s\left(\tilde{L}(x_i, x_i)\right) = (1,0)$.

The degree of reachability of the vertex x_j from the vertex x_i is determined by the expression:

$$\gamma(x_i, x_j) = \underset{k \in \overline{1,t}}{\vee} \{s(\tilde{L}_k(x_i, x_j))\} = \underset{k \in \overline{1,t}}{\vee} \{s(\mu_k, \nu_k)\}. \quad (6)$$

Here t is the number of different paths from vertex x_i to vertex x_j. The operation $\vee$ is defined according to expression (2).

If among the paths there are paths with an incommensurable degree, then as the degree of reachability we will choose the value for which the membership degree (μ_k) is the largest.

Example 1. Consider the intuitionistic fuzzy graph $\tilde{G}_1$, shown in Fig. 1.

Table 1 gives an intuitionistic fuzzy set of edges:

Table 1. Intuitionistic fuzzy set edges of graph $\tilde{G}_1$.

u_1	u_2	u_3	u_4	u_5
(0.4,0.5)	(0.6,0.4)	(0.5,0.3)	(0.2,0.7)	(0.8,0.0)

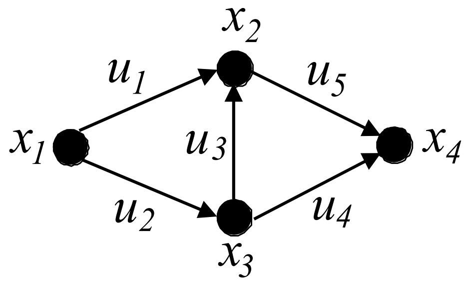

Figure 1. Intuitionistic fuzzy graph $\tilde{G}_1$

Vertex x_1 is not reachable from the vertex x_4, but the vertex x_4 is reachable from the vertex x_1 by three ways:

- $\tilde{L}_1 = (x_1, u_1, x_2, u_2, x_4)$, with degree: $s_1 = (0.4,0.5) \& (0.8,0) = (0.4,0.5)$;
- $\tilde{L}_2 = (x_1, u_2, x_3, u_4, x_4)$, with degree: $s_2 = (0.6,0.4) \& (0.2,0.7) = (0.2,0.7)$;
- $\tilde{L}_3 = (x_1, u_2, x_3, u_3, x_2, u_5, x_4)$, with degree: $s_3 = (0.6,0.4) \& (0.5,0.3) \& (0.8,0) = (0.5,0.4)$.

In this case, the degree of reachability will be defined as: $\gamma(x_1, x_4) = (0.5,0.4)$.

Example 2. Consider the intuitionistic fuzzy graph $\tilde{G}_2$, shown in Fig. 2. Table 2 gives an intuitionistic fuzzy set edges of graph $\tilde{G}_2$.

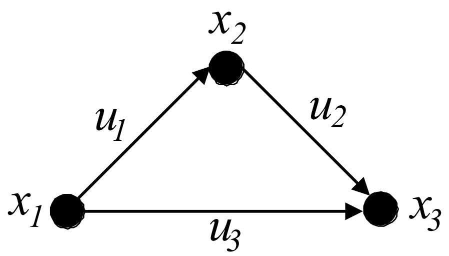

Figure 2. Intuitionistic fuzzy graph $\tilde{G}_2$

Table 2. Intuitionistic fuzzy set edges of graph $\tilde{G}_2$.

u_1	u_2	u_3
(0.8,0.1)	(0.3,0.2)	(0.5,0.3)

Vertex x_3 is reachable from the vertex x_1 by two ways with incommensurable degrees:

- $\tilde{L}_1 = (x_1, u_1, x_2, u_2, x_3)$, with degree s_1 = (0.8,0.1) & (0.3,0.2) = (0.3,0.2);
- $\tilde{L}_2 = (x_1, u_3, x_3)$, with degree s_2 = (0.5,0.3).

Therefore, the degree of reachability will be defined as: $\gamma(x_1, x_3) = (0.5,0.3)$.

Let's the number of graph vertices $|V| = n$.

Definition 1. Intuitionistic fuzzy antibase of a graph $\tilde{G}$ is a subset of vertices $\bar{B}_\beta \subseteq V$, that have the property that at least one of these vertices is reachable from any other vertices $V \backslash \bar{B}_\beta$ with an intuitionistic reachability degree of at least $\beta = (\mu_\beta, \nu_\beta)$.

Definition 2. Intuitionistic fuzzy antibase will be called *minimal* if there is no other antibase $B' \subset \bar{B}_\beta$, with the same intuitionistic reachability degree β.

Minimal intuitionistic fuzzy antibase determines the best placement of evacuation centers in the territory modeled by graph $\tilde{G}$. In this case, evacuation centers number is determined by the vertices number of of the considered antibase.

The following property follows from the definition of an intuitionistic fuzzy antibase:

Property 1. Let $\bar{B}$ a minimal intuitionistic fuzzy antibase with intuitionistic reachability degree β. Then the following statement is true:

$$(\forall x_i, x_j \in \bar{B})[\gamma(x_i, x_j) < \beta].$$

In other words, the intuitionistic reachability degree between any two vertices belonging to the minimal intuitionistic fuzzy antibase $\bar{B}$ is less than the value β of this antibase.

Proof. Assume that this is not the case, that is, there are two vertices $x_i, x_j \in \bar{B}$ such that the reachability degree $\gamma(x_i, x_j) \geq \beta$. In other words, there is some path $\tilde{L}(x_i, x_j)$ with degree $s\left(\tilde{L}(x_i, x_j)\right) \geq \beta$. Some subset of vertices (for example, Y_1) of this path belongs to the antibase $\bar{B}$, and some (for example, Y_2) does not.

An example with Y_1={y_1, y_4} and Y_2={y_2, y_3} is shown in Fig.3.

We remove the subset Y_1 and the vertex x_i from the antibase $\bar{B}$. We get a subset $\bar{B}' = \bar{B}/(Y_1 \cup \{x_i\})$, which is also an antibase with the same intuitionistic reachability degree β. That is, the antibase $\bar{B}$ is not minimal. This contradicts our assumption, which is what *Property 1* proves.

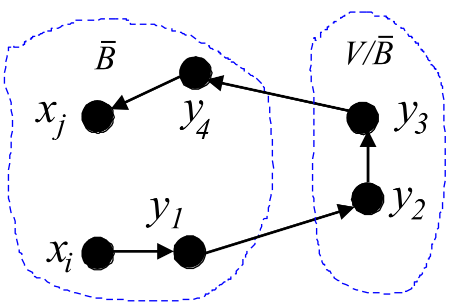

Figure 3. Example case Y_1={y_1, y_4}$\subset\bar{B}$ and Y_2={y_2, y_3}$\not\subset\bar{B}$

Consider a family of subsets of minimal intuitionistic fuzzy antibase $\Omega_i = \{\bar{B}_{i_1}, \bar{B}_{i_2}, \ldots, \bar{B}_{i_k}\}$, each of which consists of i vertices and has reachability degrees $\{\beta_{i_1}, \beta_{i_2}, \ldots, \beta_{i_k}\}$ respectively. Let β_i be the largest of these degrees. If the family $\Omega_i = \emptyset$, then $\beta_i = \beta_{i-1}$.

Definition 2. We call the intuitionistic fuzzy set

$$\tilde{B} = \{<\beta_1/1>, <\beta_2/2>, \ldots, <\beta_n/n>\}$$

the *antibase set* of the graph $\tilde{G}$ [34].

Example 3. For the intuitionistic fuzzy graph $\tilde{G}_2$, shown in Fig. 2, we have:

$$\tilde{B} = \{<(0.3,0.3)/1>, <(0.8,0.1/2>, <(1,0)/3>\}.$$

Thus, the antibase set determines the greatest possible reachability degree (β_i) for a given number of evacuation centers ($i = \overline{1,n}$).

Property 2. The following inequality holds true:

$$(0,1) \leq \beta_1 \leq \beta_2 \leq \ldots \leq \beta_n = (1,0).$$

In other words, the more vertices in the antibase, the greater the intuitionistic reachability degree.

The proof of Property 2 follows directly from the definition of the antibase set.

IV. Method for Finding Minimal Intuitionistic Fuzzy Antibases

We consider a method for finding the family of all minimal intuitionistic fuzzy antibases. This method is similar to the approach proposed in [39].

Let $\bar{B}_\beta$ be the minimal antibase with intuitionistic reachability degree β=(μ_β,ν_β). Then the following expression is true:

$$\begin{gathered}(\forall x_i \in V)[x_i \in \bar{B}_\beta \vee \\ (\exists x_j \in \bar{B}_\beta \,|\mu(x_i, x_j) \geq \mu_\beta \& \nu(x_i, x_j) \leq \nu_\beta)].\end{gathered} \tag{7}$$

For each vertex $x_i \in V$ we introduce a variable p_i such that if $x_i \in \bar{B}_\beta$ then p_i =1, and 0 otherwise.

Let us associate the intuitionistic variable ξ_{ij}=β=(μ_β,ν_β) for the expression ($\mu(x_i,x_j)$,$\gamma(x_i,x_j)$)≥β. Then, passing from the quantifier notation in expression (7) to logical operations, we obtain the truth:

$$\Phi_{\bar{B}} = \underset{i=\overline{1,n}}{\&}(p_i \vee \underset{j=\overline{1,n}}{\vee}(p_j \& \xi_{ij})).$$

Considering that $(\forall j = \overline{1,n})[\xi_{jj} = (1,0)]$, and $(\forall i = \overline{1,n})[p_i \vee \underset{j}{\vee} p_j \& \xi_{ij} = \underset{j}{\vee} p_j \& \xi_{ij}]$, the last expression will be rewritten as:

$$\Phi_{\bar{B}} = \underset{i=\overline{1,n}}{\&}(\underset{j=\overline{1,n}}{\vee}(p_j \& \xi_{ij})). \tag{8}$$

Let us open the brackets in expression (8) and reduce like terms, following the rules:

$$a \vee a \& b = a; \quad \xi_1 \& a \vee \xi_2 \& a \& b = \xi_1 \& a \text{ if } \xi_1 \geq \xi_2. \tag{9}$$

Here, $a, b \in \{0,1\}$, and $(0,1) \leq \xi_1, \xi_2 \leq (1,0)$.

Then the expression (8) can be rewritten as:

$$\Phi_{\bar{B}} = \underset{i=\overline{1,l}}{\vee}(p_{1_i} \& p_{2_i} \& \ldots \& p_{k_i} \& \beta_i). \tag{10}$$

Theorem. The variables included in each parenthesis of expression (10) define a subset of graph vertices, which is a minimum antibase set with the intuitionistic reachability degree β_i.

Proof. Let's consider that further simplification is impossible in expression (10). Let, for definiteness, disjunctive member

$$(p_1 \& p_2 \& \ldots \& p_k \& \beta), \; k < n, \; (0,1) < \beta \leq (1,0), \tag{11}$$

is included in the expression (6).

We rewrite (8) as:

$$\begin{aligned}\Phi_{\bar{B}} = \big((1,0)\&p_1 \vee \xi_{21}\&p_2 \vee \ldots \vee \xi_{n1}\&p_n\big)\& \\ (\xi_{12}\&p_1 \vee (1,0)\&p_2 \vee\vee \xi_{n2}\&p_n)\& \ldots \\ \&(\xi_{1,k+1}\&p_1 \vee \xi_{2,k+2}\&p_2 \vee \ldots \\ \vee \xi_{k,k+1}\&p_k \vee (1,0)\&p_{k+1} \vee \ldots \vee \xi_{n,k+1}p_n) \\ \&\ldots\&(\xi_{1n}\&p_1 \vee \xi_{2n}\&p_2 \vee \ldots \vee (1,0)\&p_n).\end{aligned} \tag{12}$$

Then in expression (12) the following statement should be fulfilled:

$$(\forall i = \overline{1,k})[\xi_{i,k+1} < \beta].$$

Therefore, all disjunctive members which do not contain variables $p_{k+1}, p_{k+2}, \ldots, p_n$ necessarily contain coefficients of the smaller value β in expression (10). From there, the disjunctive member (11) is not included in the expression (10). The received contradiction proves that subset $\bar{B} = \{x_1, x_2, \ldots, x_k\}$ has degree β.

We now show that the disjunctive member (11) is the minimum member. We will assume the opposite. Then should be performed condition a) or condition b):

a) There is a vertex $x \in \bar{B}$ such that $\gamma(y,x) > \beta$ holds for any vertex $y \in V/\bar{B}$. An example of such case is shown in Fig. 4.

b) There is a subset $\bar{C} \subset \bar{B}$ such that for any vertex $y \in V/\bar{C}$ there exists a vertex $x \in \bar{C}$ such that $\gamma(y,x) = \beta$. An example of such case is shown in Fig.5.

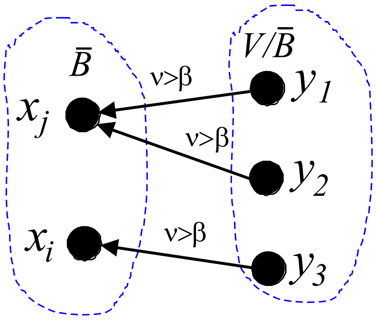

Figure 4. Example case *a)*

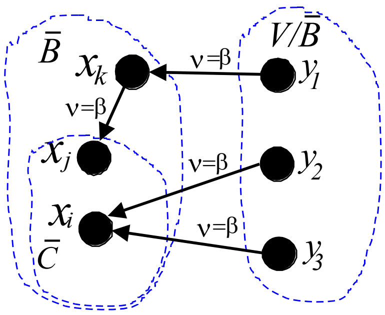

Figure 5. Example case *b)*

Let the condition a) is performed. Then the next statement is true:

$$(\forall y \in V/\bar{B})(\exists x \in \bar{B})(\gamma(y,x) = \beta' > \beta).$$

Let's present expression $\Phi_{\bar{B}}$ in the form (12). If to make logic multiplication of each bracket against each other without rules of absorption (9) we will receive n^2 disjunctive members containing exactly n elements and on one element from each bracket of decomposition (12).

We will choose one of n^2 disjunctive members as follows:

- conjunction of the pair *(1,0)&p₁* is selected from the first bracket;

- conjunction of the pair *(1,0)&p₂* is selected from the second bracket;

- etc.;

- conjunction of the pair *(1,0)&pₖ*; is selected from the k^{th} bracket;

- from $(\kappa+1)^{th}$ bracket we will select conjunction of the pair $\xi_{i_1,k+1}\&p_{i_1}$ such, that index $i_1 \in [1,k]$, and $\xi_{i_1,k+1} \geq \beta'$;

- from $(\kappa+2)^{th}$ bracket we will select conjunction of the pair $\xi_{i_2,k+2}\&p_{i_2}$, for which index $i_2 \in [1,k]$, and $\xi_{i_2,k+2} \geq \beta'$; etc.;

- from n^{th} bracket we will select conjunction of the pair $\xi_{i_{n-k},n}\&p_{i_{n-k}}$, for which index $i_{n-k} \in [1,k]$, and $\xi_{i_{n-k},n} \geq \beta'$.

Using rules of absorption (9), the received disjunctive member can be led to the form $p_1\&p_2\&\ldots\&p_k\&\beta'$, in which value $\beta' = min\{\xi_{i_1,k+1}, \xi_{k+2,i_2}, \ldots, \xi_{n,i_{n-k}}\} > \beta$ and which will be necessarily absorbed disjunctive member (11).

We obtained a contradiction, which proves the impossibility of case a).

Now suppose that condition b) is performed.

Let's for definiteness $\bar{C} = \{x_1, x_2, \ldots, x_{k-1}\}$. Considering expression $\Phi_{\bar{B}}$ in the form (12), we will choose a disjunctive member as follows:

- conjunction of the pair *(1,0)&p₁* is selected from the first bracket;

- conjunction of the pair *(1,0)&p₂* is selected from the second bracket; - etc.;

- from *(κ-1)*th bracket we will select conjunction of the pair *(1,0)&p$_{k-1}$*;

- from k^{th} bracket we will select conjunction of the pair $\xi_{i_1,k}\&p_{i_1}$ such, that index $i_1 \in [1,k-1]$, and $\xi_{i_1,k} \geq \beta$;

- from *(κ+1)*th bracket we will select conjunction of the pair $\xi_{i_2,k}\&p_{i_2}$, for which index $i_2 \in [1,k-1]$, and $\xi_{i_2,k} \geq \beta$; etc.;

- from n^{th} bracket we will select conjunction of the pair $\xi_{i_{n-k+1},n}\&p_{i_{n-k+1}}$, for which index $i_{n-k+1} \in [1,k-1]$, and $\xi_{i_{n-k+1},n} \geq \beta$.

Using rules of absorption (9), the resulting disjunctive member can be represented as $(p_1\&p_2\&\ldots\&p_{k-1}\&\beta')$, in which $\beta' = min\{\ \xi_{i_1,k}, \xi_{i_2,k+1}, \ldots, \xi_{i_{n\text{-}k+1/},n}\} \geq \beta$ and which will be necessarily absorbed by a disjunctive member (11).

We obtained a contradiction, which proves the impossibility of case b).

Theorem is proved.

V. Algorithm for Finding Minimal Intuitionistic Fuzzy Antibases

To construct the expression (10) we rewrite expression (8) like this:

$$\Phi_{\bar{B}} = \underset{i=\overline{1,n}}{\&}(\xi_{i1}\&p_1 \vee \xi_{i2}\&p_2 \vee \ldots \vee \xi_{in}\&p_n). \qquad (13)$$

Let us assign the conjunction of the pair $\xi_{ij}\&p_j$ from expression (13) to the conjunction of the pair $\xi_{ij}\&\bar{P}_j$. Here vector $\bar{P}_j = |p_i^{(j)}|$ is a binary vector that has dimension of n. Its elements are defined as:

$$p_i^{(j)} = \begin{cases} 1, \text{if } i = j \\ 0, \text{if } i \neq j. \end{cases}$$

Example 4. Let *n=4*, then the conjunction of the pair (0.5,02)*&p₂* corresponds to the binary vector $(0.5,0.2)\&\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$,

and the conjunction of the pair (0.3,0.6)&p_3 corresponds to the binary vector $(0.3,0.6)\&\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$.

The conjunction of pairs $(\xi'\&p')$ and $(\xi"\&p")$ from expression (11) corresponds the conjunction of two weighted binary vectors $\xi'\&\bar{P}'$ and $\xi"\&\bar{P}"$. Parameters ξ' and $\xi"$ take values from the $[(0,1),(1,0)]$ interval. $\bar{P}' = |p'_k|$, and $\bar{P}" = |p"_k|$, $k = \overline{1,n}$ are binary vectors. In a vector space the conjunction is defined as $(\xi'\&\bar{P}')\&(\xi"\&\bar{P}") = \xi\&\bar{P}$, here $\xi = \min\{\xi', \xi"\}$. Binary vector P' is defined as $\bar{P} = |p_k|$, $k = \overline{1,n}$, $p_k = \max\{p'_k, p"_k\}$.

Example 5. The conjunction of weighted vectors from example (4) is:

$$(0.5,0.2)\&\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\&(0.3,0.6)\&\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = (0.5,0.2)\&\begin{pmatrix}0\\1\\1\\0\end{pmatrix}.$$

We define the operation ≤ "less or equal" between binary vectors [30]. Binary vector $\bar{P}'$ is less or equal than $\bar{P}"$ if each element of $\bar{P}'$ is less or equal than the corresponding element of vector$\bar{P}"$. Formally, this looks like:

$$(\bar{P}' \le \bar{P}") \to (\forall k = \overline{1,n})(p'_k \le p"_k).$$

Example 6. $\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \le \begin{pmatrix}0\\1\\1\\0\end{pmatrix}$.

Considering the algebra in the space of weighted binary vectors, we can propose the following absorption rule:

$$\begin{gathered}(\forall\xi', \xi")(\forall\bar{P}', \bar{P}")(\xi' \ge \xi")(\bar{P}' \le \bar{P}")\\ [\xi'\&\bar{P}' \vee \xi"\&\bar{P}" = \xi'\&\bar{P}'].\end{gathered} \tag{14}$$

Example 7.

$$(0.5,0.2)\&\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \vee (0.3,0.6)\&\begin{pmatrix}1\\1\\1\\0\end{pmatrix} = (0.5,0.2)\&\begin{pmatrix}0\\1\\0\\0\end{pmatrix},$$

which corresponds to the absorption rule for elements $(0.5,0.2)\&p_2 \vee (0.3,0.6)\&p_1p_2p_3 = (0.5,0.2\_\&p_2\ )$ in the expression (13).

Now we can construct statement (12) using the conjunction operation and the rule of absorption of weighted binary vectors by the following Algorithm.

Algorithm for finding the family of all minimal intuitionistic fuzzy antibases:

Step 1°. Each element of the first bracketed (*j=1*) of expression (13) is converted to weighted binary vector. The result is to be written in the first *n* elements of the buffer vector $\overline{V_1} = \| v_i^{(1)} \|, i = \overline{1,n^2}$.

Step 2°. *j* incrementing (*j:=j+1*).

Step 3°. Each element of the bracketed expression *j* is also converted to weighted binary vectors. The result is to be written in the first *n* elements of the buffer vector $\overline{V_2} = \| v_i^{(2)} \|, i = \overline{1,n}$.

Step 4°. The next stage consists of the conjunction of two vectors $\overline{V_1}$ and $\overline{V_2}$. The result is placed into the buffer vector $\overline{V_3} = \| v_i^{(3)} \|, i = \overline{1,n^2}$. While placing elements into $\overline{V_3}$, absorption is made using rule (12).

Step 5°. All the elements of vector $\overline{V_3}$ are copied to vector $\overline{V_1}$ ($v_i^{(1)} := v_i^{(3)}, i = \overline{1,n^2}$).

Step 6°. *j:=j+1*.

Step 7°. If *j≤n* then goes to Step 3°, otherwise go to Step 8°.

Step 8°. Expression (8) is to be built using elements in the vector $\overline{V_1}$. This way we have minimal intuitionistic fuzzy antibases of graph.

Having found all minimal intuitionistic fuzzy antibases, we automatically determine the antibase set of the considered graph.

VI. EXAMPLE

Let's consider an example of the best placement of district evacuation centers, the model of which is represented by the intuitionistic fuzzy graph $\tilde{G}_3$, shown in Fig. 6. To do this, we will find all minimum antibases according to the considered approach.

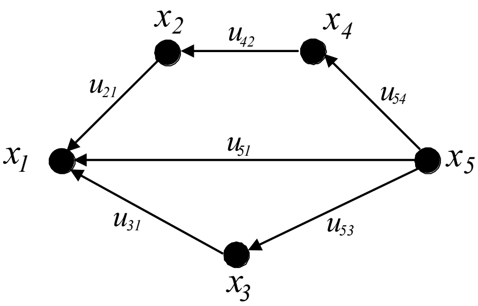

Figure 6. Intuitionistic fuzzy graph $\tilde{G}_3$

Table 3 gives an intuitionistic fuzzy set edges of graph $\tilde{G}_3$:

Table 3. Intuitionistic fuzzy set edges of graph $\tilde{G}_3$.

u_1	u_2	u_3	u_4	u_5
(0.2,0.4)	(0.3,0.1)	(0.7,0.2)	(0.8,0.1)	(0.5,0.3)

The adjacency matrix of the graph $\tilde{G}_3$ has the form:

$$R_X = \begin{array}{c} \\ x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{array}\begin{array}{c}\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5\end{array}\\ \begin{vmatrix} (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0)\\ (0.2,0.4) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0)\\ (0.5,0.3) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0)\\ (0.0,0.1) & (0.3,0.1) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0)\\ (0.2,0.6) & (0.0,0.1) & (0.8,0.1) & (0.7,0.2) & (1.0,0.0)\end{vmatrix}\end{array}.$$

Based on the adjacency matrix, one can construct reachability matrix:

To find the intuitionistic reachability matrix of a graph, we define the operation of exponentiation adjacency matrix as follows:

- zero degree adjacency matrix:

$$R_X^0 = \begin{array}{c} \\ x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{array}\begin{array}{c}\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5\end{array}\\ \begin{vmatrix} (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0)\\ (0.0,1.0) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0)\\ (0.0,1.0) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0)\\ (0.0,0.1) & (0.0,1.0) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0)\\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (1.0,0.0)\end{vmatrix}\end{array};$$

- first degree adjacency matrix: $R_X^1 = R_X$;

- second degree adjacency matrix - $R_X^2 = R_X \times R_X = |r_{ij}^{(2)}|$. Here the elements of the matrix R_X^2 are calculated according to the expression:

$$(\forall i, j = \overline{1,n})[r_{ij}^{(2)} = \bigvee_{k=\overline{1,n}} r_{ik} \& r_{kj}];$$

- t^{th} degree of matrix - $R_X^t = R_X^{t-1} \times R_X$, where the elements of the matrix R_X^2 are found similarly:

$$(\forall i, j = \overline{1,n})[r_{ij}^{(t)} = \bigvee_{k=\overline{1,n}} r_{ik}^{(t)} \& r_{kj}].$$

Note. The elements of the matrix R_X^t determine the intuitionistic degree of reachability of the vertices of the graph with the help of a path of length *t*.

Let's raise the adjacency matrix to the degree of 2, 3, …, (*n*-1). Then the intuitionistic reachability matrix R_D can be calculated as:

$$R_D = |r_{ij}^D| = \bigcup_{t=\overline{0,n-1}} R_X^t.$$

Here, the elements r_{ij}^D of the matrix R_D are defined as:

$$(\forall i, j = \overline{1,n})[r_{ij}^{(D)} = \bigvee_{t=\overline{0,n-1}} r_{ij}^{(t)}].$$

Let's find the reachability matrix for the graph shown in Fig.3. To do this, we raise the adjacency matrix to the degree 2, 3, and 4:

$$R_X^2 = \begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} & \begin{vmatrix} (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.2,0.4) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.5,0.3) & (0.3,0.2) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \end{vmatrix} \end{array};$$

$$R_X^3 = \begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} & \begin{vmatrix} (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.2,0.4) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \end{vmatrix} \end{array};$$

$$R_X^4 = \begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} & \begin{vmatrix} (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \end{vmatrix} \end{array}.$$

From here we find $R_D = \bigcup_{t=\overline{0,4}} R_X^t$:

$$R_D = \begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} & \begin{vmatrix} (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.2,0.4) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.5,0.3) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0) & (0.0,1.0) \\ (0.2,0.4) & (0.3,0.1) & (0.0,1.0) & (1.0,0.0) & (0.0,1.0) \\ (0.5,0.3) & (0.3,0.2) & (0.8,0.1) & (0.7,0.2) & (1.0,0.0) \end{vmatrix} \end{array}.$$

Using the reachability matrix R_D, we write the expression (8):

$$\begin{aligned} \Phi_{\bar{B}} = {} & [(1.0,0.0)p_1] \& [(0.2,0.4)p_1 \vee (1.0,0.0)p_2] \& \\ & \& [(0.5,0.3)p_1 \vee (1.0,0.0)p_3] \& \\ & \& [(0.2,0.4)p_1 \vee (0.3,0.1)p_2 \vee (1.0,0.0)p_4] \& \\ & \& [(0.5,0.3)p_1 \vee (0.3,0.2)p_2 \vee (0.8,0.1)p_3 \vee \\ & \vee (0.7,0.2)p_4 \vee (1.0,0.0)p_5]. \end{aligned}$$

The vectors $\bar{V}_1$ and $\bar{V}_2$ before the first iteration of the algorithm will have the following form:

$$\bar{V}_1 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}$$

and

$$\bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (1.0,0.0) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

After the first iteration of the Algorithm, we get:

$$\bar{V}_3 = \bar{V}_1 \& \bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (1.0,0.0) & (11000) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

The vectors $\bar{V}_1$ and $\bar{V}_2$ before the second iteration of the algorithm will have the following form:

$$\bar{V}_1 := \bar{V}_3 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (1.0,0.0) & (11000) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}$$

and

$$\bar{V}_2 = \begin{vmatrix} (0.5,0.3) & (10000) \\ (0.0,1.0) & (01000) \\ (1.0,0.0) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

After the second iteration of the Algorithm, we get:

$$\bar{V}_3 = \bar{V}_1 \& \bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.5,0.3) & (11000) \\ (1.0,0.0) & (11100) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

The vectors $\bar{V}_1$ and $\bar{V}_2$ before the third iteration of the algorithm will have the following form:

$$\bar{V}_1 := \bar{V}_3 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.5,0.3) & (11000) \\ (1.0,0.0) & (11100) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}$$

and

$$\bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.3,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (1.0,0.0) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

After the third iteration of the Algorithm, we get:

$$\bar{V}_3 = \bar{V}_1 \& \bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.3,0.3) & (11000) \\ (0.5,0.3) & (11010) \\ (0.3,0.1) & (11100) \\ (1.0,0.0) & (11110) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

The vectors $\bar{V}_1$ and $\bar{V}_2$ before the fourth iteration of the algorithm will look like:

$$\bar{V}_1 := \bar{V}_3 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.3,0.3) & (11000) \\ (0.5,0.3) & (11010) \\ (0.3,0.1) & (11100) \\ (1.0,0.0) & (11110) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}$$

and

$$\bar{V}_2 = \begin{vmatrix} (0.5,0.3) & (10000) \\ (0.3,0.2) & (01000) \\ (0.8,0.1) & (00100) \\ (0.7,0.2) & (00010) \\ (1.0,0.0) & (00001) \end{vmatrix}.$$

After the fourth iteration of the Algorithm, we finally get:

$$\bar{V}_3 = \bar{V}_1 \& \bar{V}_2 = \begin{vmatrix} (0.2,0.4) & (10000) \\ (0.3,0.3) & (11000) \\ (0.5,0.3) & (11010) \\ (0.3,0.1) & (11100) \\ (0.8,0.1) & (11110) \\ (1.0,0.0) & (11111) \\ (0.0,0.1) & (01000) \\ (0.0,0.1) & (00100) \\ (0.0,0.1) & (00010) \\ (0.0,0.1) & (00001) \end{vmatrix}.$$

Thus, expression (10) for graph $\tilde{G}_3$ has the form:

$$\Phi_{\tilde{B}} = (0.2,0.4)p_1 \vee (0.3,0.3)p_1p_2 \vee \\ \vee (0.5,0.3)p_1p_2p_4 \vee (0.3,0.1)p_1p_2p_3 \vee \\ \vee (0.8,0.1)p_1p_2p_3p_4 \vee (1.0,0.0)p_1p_2p_3p_4p_5 .$$

Whence it follows that this graph has 6 minimum intuitionistic antibases. From here it follows that:

- if we have 2 evacuation centers at our disposal, then the best places for their placement are the vertices x_1 and x_2;

- if we have 3 evacuation centers at our disposal, then the best places for their placement are the vertices x_1, x_2, and x_4;

- if we have 4 evacuation centers at our disposal, then the best places for their placement are the vertices x_1, x_2, x_3, and x_4;

- if we have only 1 evacuation center, then the best place is vertex x_1.

The antibase set for the considered graph $\tilde{G}_3$ will look like:

$$\tilde{B} = \{< (0.2,0.4)/1 >, < (0.3,0.3)/2 >, \\ < (0.5,0.3)/3 >, < (0.8,0.1)/4 >, < (1,0)/5 >\}.$$

This set, in particular, can help answer the question: does it make sense to use two evacuation centers, or can one be enough? In this case, the intuitionistic reachability degree will decrease from the value (0.3,0.3) to (0.2,0.4).

VII. Conclusion and Future Scope

The problem of choosing places for evacuation centers when the territory model is represented by an intuitionistic fuzzy graph was considered. To solve this problem, the definitions of the minimal antibase and the antibase set of intuitionistic fuzzy graph were introduced. The method and algorithm for calculating all minimal antibases of graph have been considered. The numerical example of finding the antibase set has been reviewed and justified. It is shown that the antibase set allows solving the problem of choosing the places of evacuation points in an optimal way, depending on the number of evacuation centers. In this paper we considered the case of optimal placement of evacuation centers at the vertices of the graph. In further studies, it is planned to consider cases of placing evacuation centers on the edges of the intuitionistic fuzzy graph. In ordinary (nonfuzzy) graphs, such a problem is related to finding absolute centers [40, 41]. This in turn leads to the need to consider the problem of generating new graph vertices.

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